



# Assessment of boundary-element method for modelling the structural response of a pavement

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Pavements: Materials, design and performance  
a, LNEC, 25 March 2010



**Road  
Materials  
and  
Pavement  
Design**



# Outline

- Pavement design – incremental procedure and modes of pavement deterioration;
- A brief description of the BEM (Boundary Element Method) formulation;
- Advantages and disadvantages of BEM;
- Application of the BEM and comparison with other methods (BISAR & ADINA);
- Future work.



# Pavement design

## Incremental design procedure

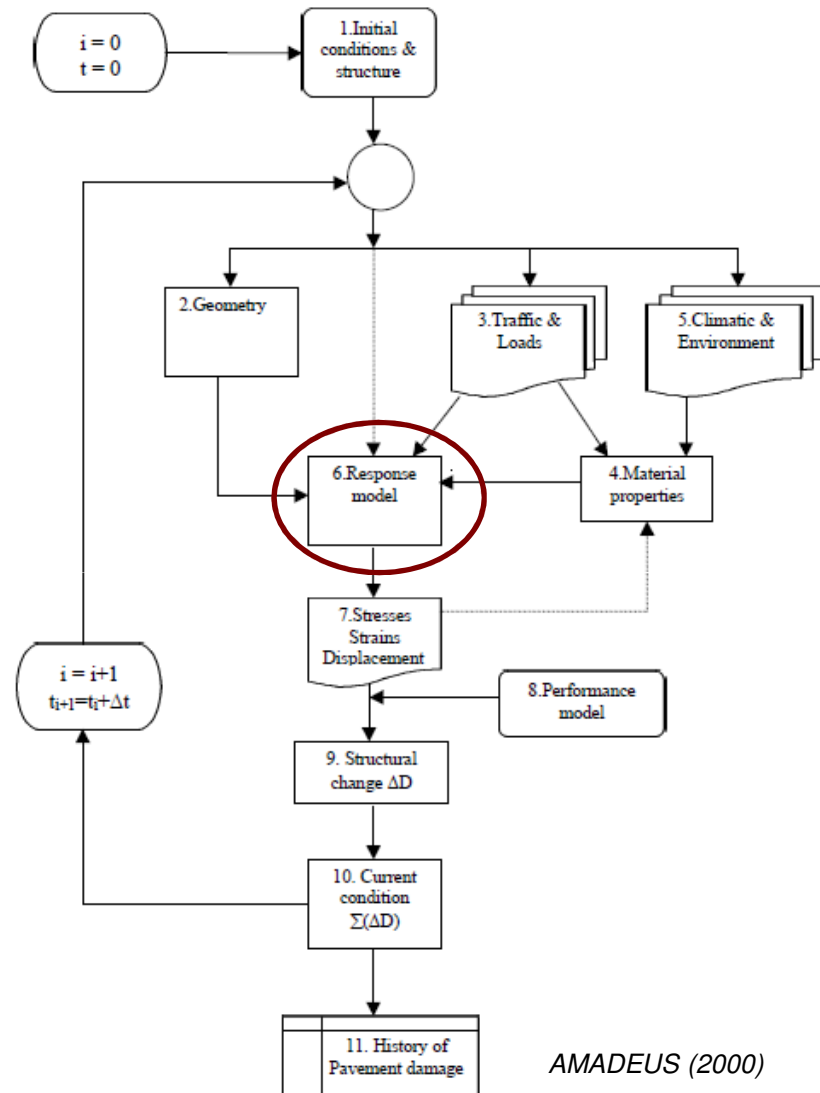
Takes into consideration the **alterations that occur during the life cycle of the pavement** (for example, material's behaviour, unevenness).

### Response model

“It is the heart of any design method” (AMADEUS, 2000).

Determines to a large extent the final result since it conditions:

- type of traffic loads (contact area)
- accuracy of response at the deterioration source
- material's behaviour
- type of response
- geometry (limited in space or not)



AMADEUS (2000)

# Pavement design - Modes of deterioration

There is a discrepancy between the actual observed modes of pavement deterioration and the modes of deterioration on which current pavement design methods are based (COST 333, 1999)

## Modes of deterioration that are generally taken for pavement design:

1. cracking from bottom of base
2. rutting in subgrade

These modes are located far below in the **ranking of the observed pavement deterioration** (result of a inquiry in the COST 333 action):

1. rutting originating in the bituminous layers
2. **cracking initiated at the surface**
3. **longitudinal unevenness**
4. loss of skid resistance
5. longitudinal cracking in the wheel path
6. cracking initiated at the bottom of the base course
7. general surface rutting
8. ravelling
9. rutting in the subgrade
10. frost heave; wear due to studded tyres; low temperature cracking



# Boundary Element Method (1/2)

## Formulation – a brief description

It is a numerical approach to solve the differential equations that describe the physical phenomena. It uses fundamental solutions which are analytically free space solutions of the governing differential equation under the action of a point source.

**Somigliana's Identity**

$$\{u(P)\} + \int_{\Gamma} [T^*(P, Q)] \cdot \{u(Q)\} \cdot d\Gamma = \int_{\Gamma} [U^*(P, Q)] \cdot \{t(Q)\} \cdot d\Gamma$$

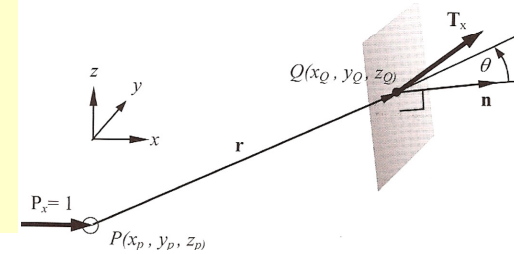
Gives the displacement at any point (P) in terms of the boundary values (Q)

**Fundamental Solutions (3D)**

$$T^*_{ij}(P, Q) = -\frac{1}{8\pi(1-\nu)r^2} \left[ \frac{\partial r}{\partial n} [(1-2\nu)\delta_{ij} + 3r_i r_j] + (1-2\nu)(n_i r_j - n_j r_i) \right]$$

$$U^*_{ij}(P, Q) = \frac{1}{16\pi G(1-\nu)r} [(3-4\nu)\delta_{ij} + r_i r_j]$$

$u_Q$  (j-direction) due to a unit load applied in P (i-direction)



When point P is very close to integration element (point Q)



Singularities ( $r \rightarrow 0$ )



Careful with evaluation of integrals to avoid errors

Taking the point P on the boundary → a **boundary integral equation** is obtained

Applying this equation at different points on the boundary



A system of equations is obtained

**which solved gives**

**Boundary values (u & t)**

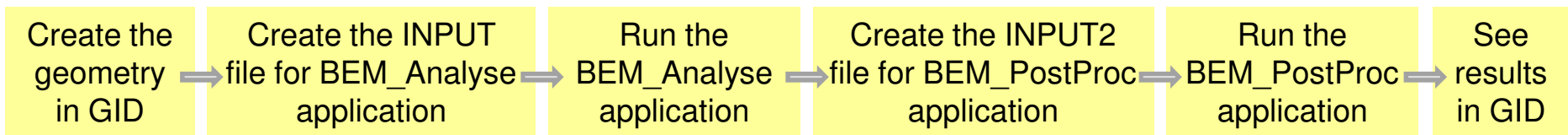


# Boundary Element Method (2/2)

## Formulation – a brief description

<p><b>Postprocessing</b></p>	<p>The values of displacements/tractions at the nodes of boundary elements are known.</p> <p>The use of fundamental solutions implies that the variation of displacement inside the domain is known in terms of boundary values.</p> $\{u(P_a)\} = \int_{\Gamma} [U^*(P_a, Q)] \cdot \{t(Q)\} \cdot d\Gamma - \int_{\Gamma} [T^*(P_a, Q)] \cdot \{u(Q)\} \cdot d\Gamma$ <p>The user may freely specify locations where results are required.</p>
<p><b>Multiple Regions</b></p>	<p>The procedure described is only applicable to homogeneous domains.</p> <p>Two approaches can be taken to deal with multi-layered systems:</p> <ol style="list-style-type: none"> <li>1. create a larger system of equations with additional unknowns at the interfaces</li> <li>2. use a similar approach to the FEM – construct a “stiffness matrix” for each layer and then assemble them.</li> </ol>

### Implementation:



# Advantages and disadvantages of BEM

<b>Advantages</b>	<b>Disadvantages</b>
<ul style="list-style-type: none"><li><input type="checkbox"/> Requires the discretization of the surface only (the dimension of the problem is effectively reduced by one)</li><li><input type="checkbox"/> There is no need to use elements on the planes of symmetry</li><li><input type="checkbox"/> Alterations of the meshes are easier</li><li><input type="checkbox"/> Gives a better accuracy in problems with stress concentration</li><li><input type="checkbox"/> It is simple and accurate in problems with infinite and semi-infinite domains</li><li><input type="checkbox"/> Provides a complete solution in terms of boundary values only</li></ul>	<ul style="list-style-type: none"><li><input type="checkbox"/> More susceptible to error when the appropriate numerical techniques are not used</li><li><input type="checkbox"/> The system of equations is non-symmetric and fully populated</li><li><input type="checkbox"/> Requires the knowledge of suitable fundamental solutions</li></ul>

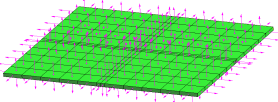
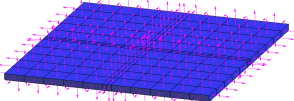
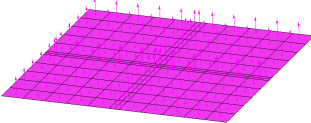


# Application (1/3)

## Structure and load definition

Two pavement structures will be considered:

- structure with thick AC layer (28 cm)
- structure with thin AC layer (12 cm)

	Modulus (MPa)	Poisson's Ratio
Asphalt Concrete layer 	2458	0.35
Granular layer (20 cm) 	200	0.30
Subgrade 	100	0.35

**Tyre:** wide single tyre - 385/65R22.5  
(tyre more used in Portugal at the portable axle)

**Tyre-pavement contact area :**

Shape: *Rectangular*

Dimension:

*Width: 22.4 cm*

*Length: 19.0 cm*

**Magnitude load: 42 kN**



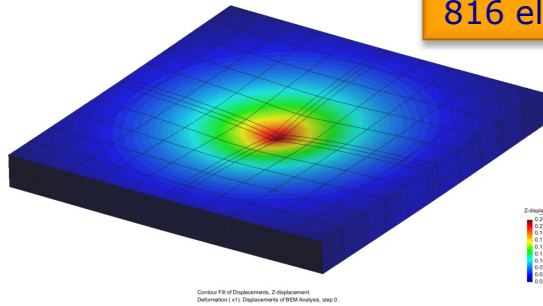
Uniform vertical stress: 987 kPa



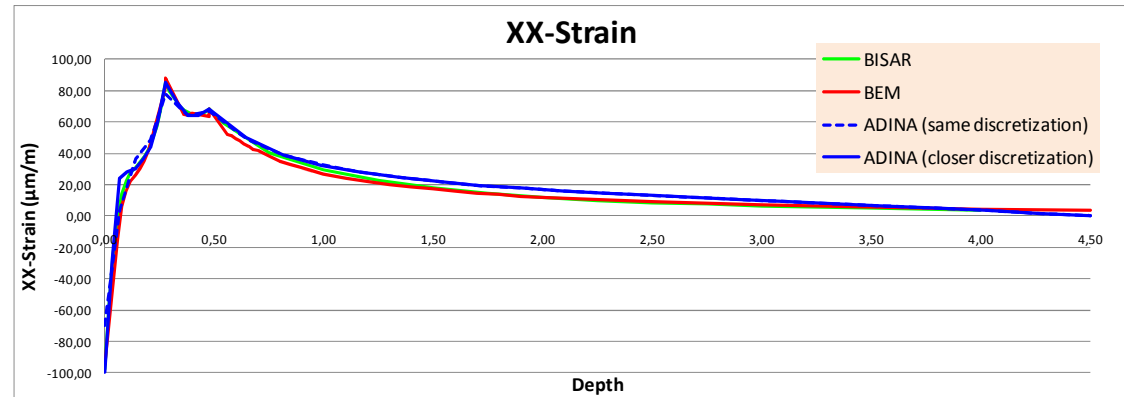
# Application (2/3) - structure with thick AC layer

BEM

1539 nodes (14%)  
816 elements (35%)

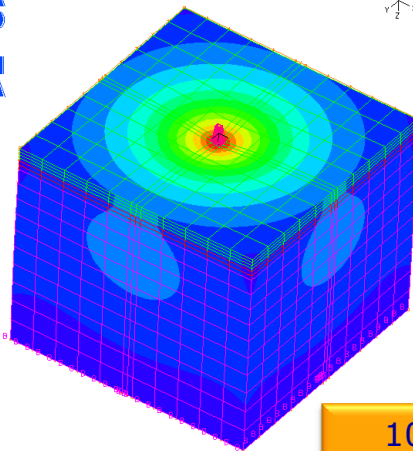


Contour Plot of Displacements, Z-displacement, Deformation (x1) Displacements of BEM analysis, step 0.

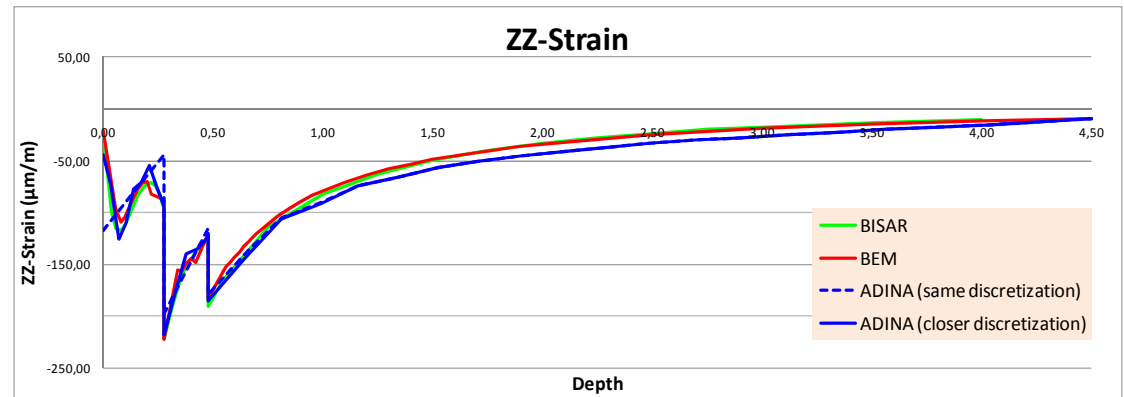


ADINA (FEM)

ADINA



10881 nodes  
2304 elements

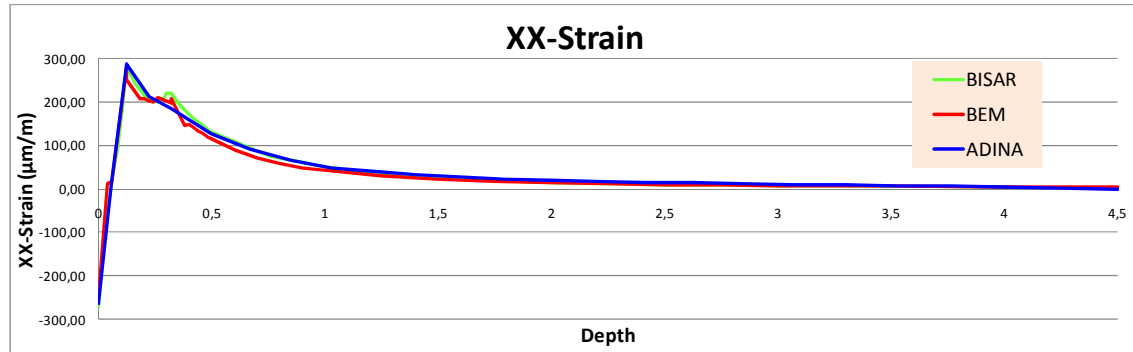
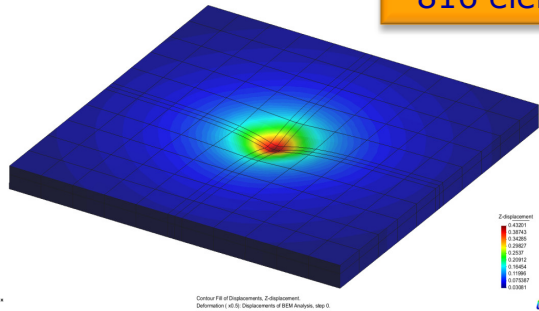




# Application (3/3) - structure with thin AC layer

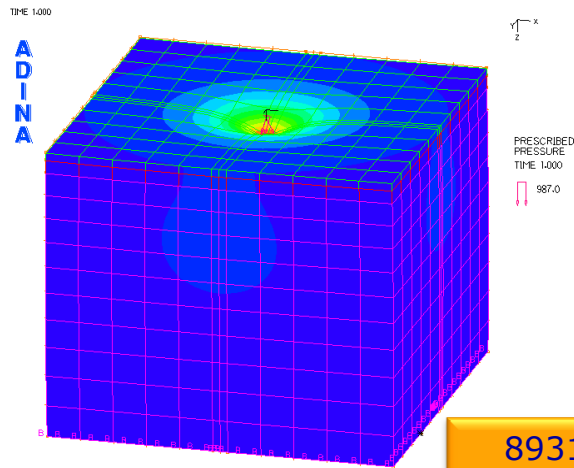
BEM

1539 nodes (17%)  
816 elements (44%)

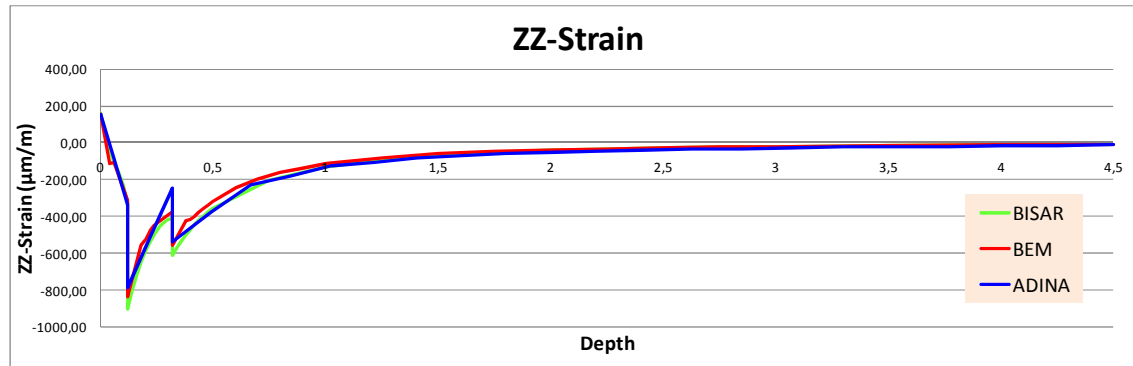


ADINA (FEM)

ADINA



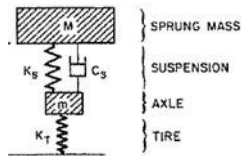
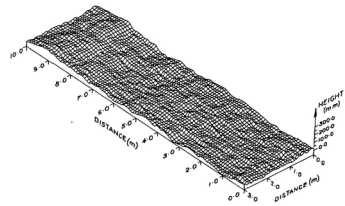
8931 nodes  
1872 elements





# Future work

For each time increment:

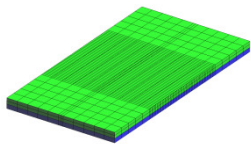


**Road surface is generated artificially** (to a target spectral density that change during the life cycle of pavement)

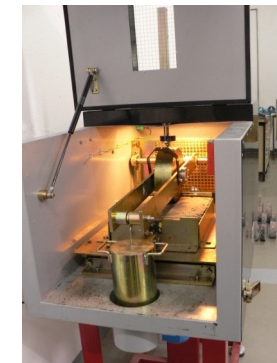
**Dynamic tyre forces are simulated**

**Loading position on the pavement surface is determined**

**Response pavement due to moving dynamic load is calculated using BEM**



Some validations and verifications of numerical results will be made by comparison with experimental results from instrumented wheel tracking tests



In future, the implementation of dual BEM will be made in order to deal with cracking initiated at the surface.

# THANK YOU